ACTUS: The algorithmic representation of financial contracts
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About this document

This document provides the technical specifications of the Algorithmic Contract Types Unified Standards (ACTUS). It is developed, maintained, and released by the ACTUS Financial Research Foundation and provided by the same to the ACTUS Users Association under the terms of the open source license with which the document is published from time to time.

Versions

This document is versioned according to the following pattern: [major].[minor]-[rc]-[revision]-[date] where [major] and [minor] are integers marking major and minor release, [rc] is an optional label "RC" indicating whether a certain [major].[minor]-release is in *candidate*-status, [revision] indicates the current revision in form of the respective git commit hash (short form), and [date] gives the respective date of the revision. Releases are recorded in the following table.

Date	Version	Description
to be released	1.0	First version of the technical specification document containing specifications for the "initial" 18 contracts.

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Contents

1. Introduction

Financial contracts are legal agreements between two (or more) counterparties on the exchange of future cash flows. Such legal agreements are defined unambiguously by means of a set of contractual terms and logic. As a result, financial contracts can be described mathematically and represented digitally as machine readable algorithms. The benefits of representing financial contracts digitally are manifold; Traditionally, transaction processing has been a field in which tremendous efficiency gains could be realized by the introduction of *machines* and machine readable contracts. Or, financial analytics by nature of the domain builds on the availability of computable representations of these agreements where for reasons of tractability often times analytical approximations are used. Recently, the rise of distributed ledger and blockchain technologies and the various use cases for *smart contracts* has opened up new possibilities for *natively digital* financial contracts.

In general, the exchange of cash flows between counterparties follows certain patterns. A typical cash flow exchange pattern is a *bullet loan* contract where principal is exchanged initially followed by cyclical interest payments and the principal is paid back (in a lump sum) at maturity of the contract. While the principal payments are fixed a variety of flavours exist for how the cyclical interest payments are determined and/or paid. As an example, interest payments may be due monthly, annually or according to arbitrary periods, they may be determined based on fixed or variable rates, different year fraction calculation methods may be used or there might be no interest due at all. Another popular pattern is that of *amortizing loans* for which, as opposed to bullet loans, principal may be paid out and paid back in portions of fixed or variable amounts and according to cyclical or custom schedules. Other types of financial contracts include but are not limited to *shares*, *forwards*, *options*, *swaps*, *credit enhancements*, *repurchase agreements*, *securitization*, etc. By focusing on the main distinguishing features, ACTUS describes the vast majority of all financial contracts with a set of about 32 generalized cash flow exchange patterns or Contract Types (CTs), respectively.

On the other hand, the legal agreements in financial contracts represent purely deterministic logic or the *mechanics of finance*, in other words. That is, a financial contract defines a fixed set of rules and conditions under which, given any external variables, the cash flow obligations can be determined unambiguously. For instance, in a *fixed rate loan* the cash flow obligations are defined explicitly. At the same time, a *variable rate loan* defines explicitly the rules under which the variable rate is fixed going forward such that the cash flow obligations can be derived unambiguously going forward. The same holds true for *derivative contracts* where the cash flow obligations arise given some underlying *reference instrument*. Similarly, for analytical purposes, given some assumption of the evolution of this reference instrument the cash flow obligations *conditioned on* this assumption can be derived unabiguously.

The properties of financial contracts described above build the foundation for a standardized, deterministic algorithmic description of the cash flow obligations arising from such agreements. Thereby, this description is technology agnostic and supports all use cases necessary for this very standard to be used throughout all finance functions from front office to back office and covering pricing, deal origination, transaction processing, as well as analytics, in general, and liquidity projections, valuation, P&L calculations and projections, and risk measurement and aggregation, in particular. Furthermore, this standard builds a formidable basis for distributed ledger-powered, natively digital *financial state machines* or *smart contracts*, in other words.

2. Financial contract taxonomy

Below an overview of the ACTUS Contract Types taxonomy.

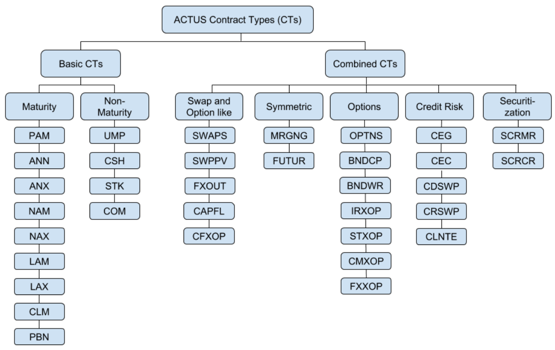

Figure 1. An overview of the ACTUS Contract Types Taxonomy.

Table 1 provides further details on the ACTUS Taxonomy including the real-world financial contracts covered through the various ACTUS Contract Types.

Family	**Class**	**Type**	**Description**	**Covered contracts**
Basic CT	Maturity	PAM: Principal at Maturity	Principal payment fully at Initial Exchange Date (IED) and repaid at Maturity Date (MD). Fixed and variable rates.	All kind of bonds, term deposits, bullet loans and mortgages etc.
		ANN: Annuity	Principal payment fully at IED and interest plus principal repaid periodically in constant amounts till MD. If variable rate, total amount for interest and principal is recalculated to be fully matured at MD.	Classical level payment mortgages, leasing contracts etc.

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Family	Class	Type	Description	Covered contracts
		NAM: Negative Amortizer	Similar as ANN. However when resetting rate, total amount (interest plus principal) stay constant. MD shifts. Only variable rates.	Special class of ARMs (adjustable rate mortgages), Certain loans.
		LAM: Linear Amortizer	Principal payment fully at IED. Principal repaid periodically in constant amounts till MD. Interest gets reduced accordingly. If variable rate, only interest payment is recalculated. Fixed and variable rates.	Many amortizing loans
		ANX: Exotic Annuity	Exotic version of ANN However step ups with respect to (i) Principal, (ii) Interest rates are possible. Highly flexible to match totally irregular principal payments. Principal can also be paid out in steps.	A special version of this kind are teaser rate loans and mortgages with annuity features
		LAX: Exotic Linear Amortizer	Exotic version of LAM. However step ups with respect to (i) Principal, (ii) Interest rates are possible. Highly flexible to match totally irregular principal payments. Principal can also be paid out in steps.	A special version of this kind are teaser rate loans and mortgages
		NAX: Exotic Negative Amortizer	Exotic version of NAM However step ups with respect to (i) Principal, (ii) Interest rates are possible. Highly flexible to match totally irregular principal payments. Principal can also be paid out in steps.	A special version of this kind are teaser rate loans and mortgages with variable MD
		CLM: Call Money	Loans that are rolled over as long as they are not called. Once called it has to be paid back after the stipulated notice period.	Interbank loans with call features
		PBN: Perpetual Bonds	Bonds without any maturity date. Interest is paid into eternity if is not terminated.	Consoles, war loans
	Non-Maturity	UMP: Undefined Maturity Profile	Principal paid in and out at any point in time without prefixed schedule. Interest calculated on outstanding and capitalized periodically. Needs link to a behavioral function describing expected flows.	Saving products of all kind, current accounts. In some countries even variable rate mortgages can be represented with this CT
		CSH: Cash	Cash or cash equivalent position	Cash, deposits at central bank
		STK: Stock	Any instrument which is bought at a certain amount (market price normally) and then follows an index.	All straight stocks
		COM: Commodity	This is not a financial contract in its proper sense. However it tracks movements of commodities such as oil, gas or even houses. Such commodities can serve as underlyings of commodity futures, guarantees or simply asset positions.	Oil, gas, electricity, houses etc.
Combined	Swap and Option like	SWAPS: Swap	Exchange of two basic CTs (PAM, ANN etc.). Normally one is fixed, the other variable. However all variants possible including different currencies for cross currency swaps, basic swaps or even different principal exchange programs.	All kind of swaps. The variety is defined by the underlying CTs which currently are PAM and ANN in all tis flavors. With each new basic CT the variety rises
		SWPPV: Plain Vanilla Swap	Plain vanilla swaps where the underlying is always a PAM and one leg is fixed, the other variable. Plain vanilla cross currency swaps also covered.	More than 90% of all interest rate swaps follow this simple pattern.
		FXOUT: Foreign Ex-change Outright	Two parties agree to exchange two fixed cash flows in different currencies at a certain point in time in future.	Any FX-outright transaction. This is also the underlying of FX-options and FX futures
		CAPFL: Cap Floors	Interest rate option expressed in a maximum or minimum interest rate	Caps and Floor options

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Family	Class	Type	Description	Covered contracts
		CFXOP: Exotic Cap Floor	Exotic variants of caps and floors	
	Securitization	SCRMR: Securitized Instruments Market Risk	Instruments bundled and traded in tranches without any specific credit risk feature	MBS, ABS, Principal only, Interest only instruments
		SCRCR: Securitized instrument Credit Risk Feature	Instruments bundled and traded in tranches that include specific credit risk feature	CDOs
	Symmetric	MRGNG	A generic margining contract governing the agreement of margining usually present at central clearing houses	
		FUTUR: Future	Keeps track of value changes for any basic CT as underlying (PAM, ANN etc. but also FXOUT, STK, COM). Handles margining calls.	Standard interest rate, FX, stock and commodity futures.
	Options	OPTNS: Option	Calculates straight option pay-off for any basic CT as underlying (PAM, ANN etc.) but also SWAPS, FXOUT, STK and COM. Single, periodic and continuous strike is supported.	European, American and Bermudan options with Interest rate, FX and stock futures as underlying instruments
		BNDCP: Callable or puttable maturity contract	Bonds with a call or put option. If option is exercised, underlying bond ceases to exist.	Callable and puttable bonds or loans
		BNDWR: Bond with warrant	Bonds with a warrant. If option is exercised, underlying bond continues to exist.	Warrants
		IRXOP: Exotic Interest Rate Option	Exotic interest rate options	
		STXOP: Stock Option	Exotic stock options	
		CMXOP: Exotic Commodity Option	Exotic commodity options	
		FXXOP: Exotic FX Option	Exotic FX options	
	Credit Risk	CEG: Guarantees	Guarantee is a credit enhancement contract. It creates a relationship between a guarantor, an obligee and a debtor, moving the exposure from the debtor to the guarantor.	Personal guarantee. Government guarantee. Underlyings of CDOs.
		CEC: Collateral	Collateral is a credit enhancement contract. It creates a relationship between a collateral an obligee and a debtor, covering the exposure from the debtor with the collateral.	Mortgages include a collateral contract. Any coverage with financial or physical collateral
		CDSWP: Credit Default Swap	All sorts of credit default swaps	

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Family	Class	Type	Description	Covered contracts
		CRSWP: Total Return Swap	All sorts of total return swaps	
		CLNTE: Credit Linked Note	All sorts of credit linked notes	

Table 1: Financial contract taxonomy

3. Notations

3.1. **Contract Attributes.** Contract Attributes (attributes) represent the legal contractual terms that define the exchange of cash-flows of a financial contract. Attributes are introduced by the ACTUS data standard and described in the ACTUS Data Dictionary (DD). Different data types exist and are defined in the DD. In particular, scalar-type and vector-type attributes are defined. Throughout this document the attribute short name is used according to the DD. Further, vector-type attributes may be indexed with a subscript indicating that a specific vector-element is referenced.

Example 1 (Contract Attribute)**.** *The ACTUS attribute Initial Exchange Date is represented in short form* `IED`*.*

Example 2 (Element of Vector-Type Attribute)**.** *The ACTUS attribute Array Cycle Anchor Date of Principal Redemption is a vector-type attribute and represented as* `ARPRANX`*. The i-th element of the vector is referenced by* $\texttt{ARPRANX}_i$*.*

3.2. **∅-Operator.** The ∅-operator is used to indicate that a certain property is undefined or, in other words, that no value has been assigned to the respective property. In particular, for optional contract attributes it means that the attribute is not defined and for schedule times (see section 4.1) it means that the respective schedule is empty, i.e. no schedule time defined.

Example 3 (Undefined Attribute)**.** $\texttt{IPANX} = \varnothing$ *indicates that attribute* `IPANX` *is undefined.*

Example 4 (Empty Schedule)**.** $\vec{t}^{\,IP} = \varnothing$ *means the same as* $\vec{t}^{\,IP} = \{\}$*, with* $\{\}$ *the empty set, and states that the IP schedule* $\vec{t}^{\,IP}$ *does not contain a schedule time.*

3.3. t_0**-Time.** t_0 marks the time as per which the state of a contract is represented in form of the respective set of attributes. Status Date `SD` itself is an attribute of the contract. In general, from the contractual logic we are able to derive any contractual events and contract states for any time $t > t_0$.

3.4. **State Variables.** State Variables describe the inner state of a financial contract at a certain point in time t during its lifetime such as (outstanding) Nominal Value, applicable Interest Rate, or the contract performance through Contract Status. State Variables are written in the short form as defined in table 3 with first letter capitalized, printed in bold, and indexed with time.

Example 5 (State Variables)**.** $\boldsymbol{Nvl}_t$ *refers to the State Variable Nominal Value observed at per time* t*.*

3.5. **Contract Events.** A contract event e_t^k refers to any contractually scheduled or un-scheduled event at a certain time t and of a certain type k. Contract events mark specific points in time during the lifetime of a financial contract at which a cash flow is being exchanged (see section 3.7) or the State Variables of the contract are being updated (see section 3.6). Contract Events types k are written in the short form as defined in table 4.

As an event always has an associated event time t and payoff $c \in \mathbb{R}$ we define two operators allowing to retrieve these quantities for any single event e_t^k or set of events $\{e_t^k, e_s^j, ...\}$ as follows;

$$\tau(x) = \begin{cases} t & \text{if} \quad x = e_t^k \\ \{t, s, ...\} & \text{else if} \quad x = \{e_t^k, e_s^j, ...\} \end{cases}$$

$$f(x) = \begin{cases} c & \text{if} \quad x = e_t^k \\ \{c_1, c_2, ...\} & \text{else if} \quad x = \{e_t^k, e_s^j, ...\} \end{cases}$$

with $c_1 = f(e_t^k), c_2 = f(e_s^j), ...$

Example 6 (Contract Events)**.** *The Initial Exchange Date event with event time* s *is written as* e_s^{IED} *with* $\tau(e_s^{IED}) = s$ *and* $f(e_s^{IED}) = c$ *where for any contract type* `CT` $c = POF\_IED\_\texttt{CT}()$*.*

3.6. **State Transition Functions.** State Transition Functions (STF) define how the State Variables are being updated when a certain Contract Event e_t^k applies from a pre-event (i.e. pre-time t) state indexed t^- to a post-event (i.e. post-time t) state indexed t^+. These functions are specific to a certain Contract Event and Contract Type. STFs are written according to the following pattern STF_[event type]_[contract type]() where [event type] and [contract type] refer to the respective event type and contract type to which the STF belongs.

Example 7 (State Transition Functions)**.** *The STF for an IP event and PAM contract is written as STF_IP_PAM() and maps e.g. state variable Nominal Accrued from a pre-event state* $\boldsymbol{Nac}_{t^-}$ *to post-event state* $\boldsymbol{Nac}_{t^+}$*.*

3.7. **Payoff Functions.** Payoff Functions (POF) define how the cash flow $c \in \mathbb{R}$ for a certain Contract Event e_t^k is being derived from current State Variables and Contract Attributes. If necessary, the resulting cash flow can be indexed with the event time c_t. These functions are specific to a certain Contract Event and Contract Type. POFs are written according to the following pattern POF_[event type]_[contract type]() where [event type] and [contract type] refer to the respective event type and contract type to which the STF belongs.

Example 8 (Payoff Functions)**.** *The POF for an IP event* e_t^{IP} *and PAM contract is written as POF_IP_PAM() with* $f(e_t^{IP}) = POF\_IP\_PAM()$.

3.8. **Date/Time.** ACTUS builds on the ISO 8601 date/time format. Hence, dates are generally expressed in the following format: [YYYY]-[MM]-[DD]T[hh]:[mm]:[ss]. Time zone information is currently not supported.
A special case is *midnight*. ISO 8601 recognizes both times 00:00:00 and 24:00:00 each referring to midnight. Yet, while 24:00:00 refers to the end of one day, 00:00:00 refers to the beginning of the following day. In ACTUS the interpretation is the same why the time period (measured in any time unit) between the two points in time will always be zero.
For brevity, we use the term *time* for a specific date-time variable and in particular abbreviation Tev for the date of a Contract Event.

3.9. **Event Sequence.** Contract Events of different types may occur at the same time, i.e. exactly the same point in time. In this case, the sequence of evaluating their State Transition and Payoff Functions is decisive for the resulting cash flows and state updates. The Event Sequence given for all events in table 4 defines the order in which these functions are evaluated for the respective event types.

3.10. **Contract Lifetime.** The lifetime of an ACTUS contract is the time period of its existence from the perspective of the analyzing user. For every point in time during its lifetime, an ACTUS contract can be analyzed in terms of current State Variables and future cash flows.

The lifetime of a contract starts with its `SD` and ends with $\min(MD, AMD, PR^*, STD, TD, t^{max})$.

Note that PR^* refers to the PR event of a maturity contract after which **Nvl**=0.0 (i.e. at which the remaining outstanding principal is redeemed). Further, `MD`, `AMD`, and PR(**Nvl**=0.0) in the definition above do only apply for maturity contracts but have to be considered infinity in all other cases. Similarly, `STD` only applies for certain contracts and is considered infinity for all others. Finally, t^{max} is a parameter that may be used to restrict the considered lifetime in an analysis. In particular, this parameter is used for contracts that do not have a *natural* end to their lifetime such as STK.

4. Utility Functions

4.1. **Schedule.** A schedule is a function S mapping times s, T with $s < T$ and cycle c onto a sequence $\vec{t}$ of cyclic times

$$S(s,c,T) = \vec{t} = \begin{cases} \{\} & \text{if} \quad s = \varnothing \wedge T = \varnothing \\ s & \text{else if} \quad T = \varnothing \\ (s,T) & \text{else if} \quad c = \varnothing \\ (s = t_1, ..., t_n = T) & \text{else} \end{cases}$$

with $t_i < t_{i+1}, i = 1, 2, ...$ While the schedule function can be used to create arbitrary sequences of times, it is usually used to generate sequences of cyclic events $\vec{t}^k$ of a certain type k, e.g. $k = IP$ for interest payment events (cf. table 4) and the following build inputs to the function

- s = k`ANX` with k`ANX` attribute cycle anchor date of event type k
- c = k`CL` with k`CL` event type k's schedule cycle
- T = `MD` with `MD` the contract's maturity

Thereby, cycles k`CL` have format NPS where

- N is an integer
- P is a time period unit (D=Day, W=Week, M=Month, Q=Quarter, H=Half Year, Y=Year)
- S is a *stub* information (+=long last stub, −=short last stub)

Further, the last stub is defined as follows

- if $t_{n-1} + c = T \vee S =$'-' then no stub correction applies
- else t_n is removed from the schedule

The sequence of schedule times $\vec{t}^k$ may also be influenced by the `EOF` and `BDC` conventions and the full function syntax becomes $S(s, c, T, \texttt{EOMC}, \texttt{BDC})$. Due to such effects the sequence of schedule times can be non-equidistant or, in other words, $t_i^k - t_{i-1}^k \neq t_j^k - t_{j-1}^k, i \neq j$.

Note that for brevity we will omit the `EOMC` and `BDC` function arguments throughout this document.

4.2. **Array Schedule.** Array Schedules are defined by vector-valued inputs $\vec{s} = (s_0, s_1, ..., s_m)$ and $\vec{c} = (c_0, c_1, ..., c_m)$ to the regular schedule function

$$S(\vec{s}, \vec{c}, T) = (S(s_0, c_0, s_1 - c_0), S(s_1, c_1, s_2 - c_1), ..., S(s_m, c_m, T))$$

Hence, array schedules are a generalization for regular schedules which coincide for $m = 1$. In accordance with regular schedules `EOMC` and `BDC` conventions also apply here.

4.3. **End Of Month Shift Convention.** For schedules $\vec{t}^k$ starting at time s which marks the end of a month with 30 or less days, e.g. April 30, and with a cycle c being a multiple of 1M- attribute `EOM` defines whether the schedule times are to fall on the 30th of all months (same day) or the 31st (end of month).

More specifically, `EOM` has an effect on a schedule $\vec{t}^k$ only if:

- s is the last day of a month with less than 31 days (Feb, April etc.)
- c = NPS with $P \in (M, Q, HorY)$

As per the DD `EOM` can take one of the following values:

- EOM (EndOfMonth): times $t_i, i = 1, 2, ..., n-1$ are moved to the end of the respective months
- SD (SameDay): times $t_i, i = 1, 2, ..., n-1$ remain unchanged except in February, where it will go to the last day if the day of month of time s is higher than the number of days of February

4.4. **Business Day Shift Convention.** In general, contract events are scheduled for business days only. Therefore, the `BDC` convention defines how scheduled times $t_i, i = 1, 2, ..., n-1$ are shifted in case they fall on a non-business day:

NULL: No shift

- SCF: Shift/Calculate following: The event is shifted to the following non working day. Calculation of the event happens after the shift
- SCMF: Shift/Calculate modified following: The event is shifted to the following non working day. However, if the following day happens to fall into the next month, then take preceeding non-working day. Calculation of the event happens after the shift
- CSF: Calculate/Shift following: Same like SCF however calculation of the event happens before the shift
- CSMF: Calculate/Shift modified following: Same like SCMF however calculation of the event happens before the shift
- SCP: Shift/Calculate preceding: The event is shifted to the last preceding non working day. Calculation of the event happens after the shift
- SCMP: Shift/Calculate modified preceding: The event is shifted to the last preceding non working day. However, if the preceding day happens to fall into the previous month, then take next non-working day. Calculation of the event happens after the shift
- CSP: Calculate/Shift preceding: Same like SCP however calculation of the event happens before the shift
- CSMP: Calculate/Shift modified preceding: Same like SCMP however calculation of the event happens before the shift

4.5. **Business Day Calendar.** Whether a specific day is a business day (cf. previous section) is defined by attribute `CLDR`. Such conventions generally depend on regional official holiday calendars. The Business Day Function interface allows determining for some `CLDR` whether any time t is a business day or not

$$B : t \mapsto \{true, false\}$$

where *true* indicates that t is a business day and $false$ that it is a holiday.

Example 9. *Two standard* `CLDR` *implementations are the following*

- *NoHoliday (default): every calendar day is a business day*
- *MondayToFriday: all weekdays Monday, Tuesday, Wednesday, Thursday, and Friday are business days*

4.6. **Year Fraction Convention.** Interest income and other calculations are based on *per annum* interest rates. Therefore, the year-fraction function interface Y is used to calculate the *fraction of a year* between any two times s and t with $t > s$ for which e.g. an (per annum) interest rate applies according to some day count convention `DCC`

$$Y : s, t, \texttt{DCC} \mapsto \mathbb{R}$$

Note, the year fraction function interface only defines the structure of year fraction functions but not an actual implementation thereof, or the respective `DCC`, respectively. Therefore, any `DCC` can be implemented according to the interface above supporting user-defined year fraction functions.

For brevity we will omit the `DCC` function argument wherever this does not lead to confusion.

4.7. **Contract Role Sign Convention.** The two counterparties to a financial contract are defined through attributes `LEIRC` and `LEICP`. The first is the party initially *creating* the contract and the second is the counterparty, respectively. Thereby, both `LEIRC/LEICP` can take any *role* in the contract or, more specifically, they can be the lender or borrower in a loan (PAM), fixed receiver or payer in an interest rate swap (SWAPS), etc.

The *role* of the `LEIRC` is defined through attribute `CNTRL`. The *role* of `LEICP` is derived as the *opposite* side to the contract. Apart from `CNTRL` the attributes are *neutral* to the *role* of `LEIRC` (or `LEICP`).

On the other hand, contractual cash flows generated by the POFs and certain state variables are *role-sensitive*. That is, from the perspective of the `LEIRC` these quantities represent either claims or obligations. Contract Role Sign function R maps the `CNTRL` attribute into $+1$ indicating a claim or -1 indicating an obligation

$$R : \texttt{CNTRL} \to \{-1, +1\}$$

When multiplying with a cash flow x the Contract Role Sign function thereby defines the direction of that flow:

$x > 0$: x flows from `LEICP` to `LEIRC`
$x < 0$: x flows from `LEIRC` to `LEICP`

Table 2 defines the domain of the Contract Role Sign function, i.e. the range of attribute `CNTRL`, with meaning and Contract Role Sign to which the function maps.

Value	**Meaning**	**R**
RPA	Real position asset	+1
RPL	Real position liability	-1
CLO	Role of a collateral	+1
CNO	Role of a close-out-netting	+1
COL	Role of an underlying to a collateral	+1
LG	Long position	+1
ST	Short position	-1
BUY	Protection buyer	+1
SEL	Protection seller	-1
RFL	Receive first leg	+1
PFL	Pay first leg	-1
RF	Receive fix leg	+1
PF	Pay fix leg	-1

Table 2. Contract Role definitions.

4.8. **Contract Default Convention.** Performance of a contract indicates whether as per a certain time all parties involved adhere to their obligations arising from the contract. Attribute `CTS` captures a contract's performance as per t_0. For any time $t > t_0$ and depending on the behavior of the parties involved the contract can migrate into different contract (performance) statuses from *'performing'* to *'default'*. State variable $\mathbf{Prf}_t$ (cf. table 3) captures these dynamics and the performance as per any time $t > t_0$.

The Contract Default Convention is a function D that maps the $\mathbf{Prf}_t$ state variable into +1 indicating that the contract is performing or 0 which reflext default and, from an analytical perspective, means that future cash flows *cancel out*:

$$D(\mathbf{Prf}_t) = \begin{cases} 1 & \text{if} \quad \mathbf{Prf}_t \neq \text{'D'} \\ 0 & \text{else} \end{cases}$$

4.9. **Annuity Amount Function.** In an *Annuity* contract (ANN) the annuity amount is paid regularly from the *borrower* to the *lender*. Thereby, the annuity amount is comprised of a principal repayment portion and an interest portion and and dimensioned such that the total nominal amount n at time t is fully repaid at maturity T of the annuity. The Annuity Amount function A(,c,o,m,p)utes the annuity amount as follows

$$A(s,T,n,a,r) = (n+a)\frac{\prod_{i=1}^{m-1} 1 + rY(t_i, t_{i+1},)}{1 + \sum_{i=1}^{m-1} \prod_{j=i}^{m-1} 1 + rY(t_j, t_{j+1})}$$

with a the accrued interest as per time s, r the actual interest rate, $t_i, i = 1, 2, ..., m$ the schedule times $\inf t, t \in \vec{t}^{PR} \wedge t > s$, m the number of times t_i, and $\vec{t}^{PR}$ the PR-event schedule times of the Annuity contract as described in section 9.5.

5. Contract State Variables

Driven by Contract Events (see section 3.5) certain contractual dimensions, state variables, of financial contracts may change during the lifetime of a financial contract. Thereby, the set of State Variables varies for different CTs. Table 3 represents the set of all covered State Variables throughout the universe of CTs.

By definition, State Variables are updated through Contract Events only. The value of State Variables always shows the state of the contract **after** the respective Contract Event.

Name	**Abbrv.**	**Explanation**
Performance	$\mathbf{Prf}_t$	Contract performance
Last Event Date	$\mathbf{Led}_t$	The date of the most recent Contract Event
Nominal Value	$\mathbf{Nvl}_t$	The outstanding nominal value
Secondary Nominal Value	$\mathbf{Nv2}_t$	The outstanding nominal value of the second leg
Nominal Rate	$\mathbf{Nrt}_t$	The applicable nominal rate
Nominal Accrued	$\mathbf{Nac}_t$	The current value of nominal accrued interest at the Nominal Rate
Interest Calculation Base	$\mathbf{Icb}_t$	The basis at which interest is being accrued if different from $\mathbf{Nvl}_t$
Notional Scaling Multiplier	$\mathbf{Nsc}_t$	The multiplier being applied to Notional/Principal related cashflows
Interest Scaling Multiplier	$\mathbf{Isc}_t$	The multiplier being applied to Interest related cash-flows
Next Principal Redemption Payment	$\mathbf{Npr}_t$	The value at which $\mathbf{Nvl}_t$ is being repaid. This may be including or excluding of interest depending on the instrument
Payoff at Settlement	$\mathbf{Pos}_t$	The payoff of the contract if fixed at time t. If evaluated during the lifetime of the contract this quantity gives a hypothetical payoff (e.g. for an OPTNS contract it defines whether the option is in-the-money or not).

Table 3. State variables

6. Contract Event Types

An overview and description of various event types can be found in table 4.

Type	**Name**	**Explanation**	**Seq.**
IED	Initial Exchange Date	Scheduled date of first principal event, start of accrual calculation	1
IPCI	Interest Capitalization	Scheduled interest payment which is capitalized instead of paid out	2
IP	Interest Payment	Scheduled interest payment	3
FP	Fee Payment	Scheduled fee payment	4
PR	Principal Redemption	Scheduled principal redemption payment	5
PI	Principal Increase	Scheduled principal increase payments	6
PRF	Principal Payment Amount Fixing	Scheduled re-fixing of principal payment (PR or PI) amount	7
PY	Penalty Payment	Payment of a penalty (e.g. due to early repayment of principal outstanding)	8
PP	Principal Prepayment	Unscheduled (early) repayment of principal outstanding	9
CD	Credit Default	Credit event of counterparty to a contract	10
RRF	Rate Reset Fixed	Scheduled rate reset event where new rate is already fixed	11
RR	Rate Reset Variable	Scheduled rate reset event where new rate is fixed at event time	12
DV	Dividend Payment	Scheduled (e.g. announced) dividend payment	13
PRD	Purchase Date	Purchase date of a contract bought in the secondary market	14
MR	Margin Call Date	Scheduled margin call event	15
TD	Termination Date	Sell date of a contract sold in the secondary market	16

Continued on next page

Continued from previous page

Type	Name	Explanation	Seq.
SC	Scaling Index Revision	Scheduled re-fixing of a scaling index	17
IPCB	Interest Payment Calculation Base	Scheduled update to the calculation base for IP accruing	18
XD	Execution Date	Scheduled or unscheduled execution of e.g. an OPTNS or FUTUR contract	19
STD	Settlement Date	Date when payment for derivatives is settled	20
MD	Maturity Date	Scheduled maturity or expiry of a contract	21
AD	Analysis Event	Retrieves current contract states without alter these	22

Table 4: Contract Events Definitions.

7. Risk Factor Observer

The payoff of financial contracts always depends on the context in which it is evaluated and which is comprised of the following dimensions; counterparties, markets, and behavioral factors. We refer to these as the *risk factors* to which financial contracts are exposed to. This indicates that these factors are source of uncertainty because financial contracts only reference the factors but their dynamics is outside the control of any contractual agreement. Thus, such factors have to be *observed* and their changing states accounted for when evaluating the payoff of financial contracts. Therefore, we consider a standardized interface $O^o(i,t,S,M)$ that allows for *observing*; (1) the state of a certain risk factor i at any time t if $o=$'rf'

$$O^{rf}: i,t,S,M \mapsto \mathbb{R}$$

and (2) contractual but non-scheduled events if $o=$'ev'

$$O^{ev}: i,t,S,M \mapsto \{e^k_t, e^l_s, ...\}$$

The parameters to the Risk Factor Observer interface are as follows:

- i : the identifier of the risk factor *observed*
- t : the time for which the risk factors state should be evaluated
- S : the inner states of the contract at time t
- M : the contract terms of the contract as per time t

Note that the observer interface only defines the structure of an actual observer function but not the actual implementation. Thus, the interface allows for user-defined implementations of observer functions allowing e.g. for representing arbitrary assumptions on the evolution of future risk factor states which is key for any type of forward-looking analysis.

Example 10 ('rf'-Observer)**.** *The market-driven 3-month USD-Libor reference rate used as the variable rate in a variable rate loan contract is observed at any time t through* $O^{rf}(\texttt{MarketObjectCodeRateReset}, t)$.

Example 11 ('rf'-Observer)**.** *Unscheduled (pre-) repayments of outstanding notional in a mortgage contract is observed at any time t through* $O^{ev}(\texttt{CID}, t)$.

For brevity we will omit the S and M function arguments wherever this does not lead to confusion.

8. Child Contract Observer

The payoff of certain contracts, i.e. of *Combined Contracts* according to the taxonomy in 1, is derived from certain quantities of child contracts. Often times, such contracts are referred to as *underlying instruments* or simply *underlyers* as they build the basis for the payoff of the *parent* contract. Therefore, we consider a standardized interface U^o that allows for *observing* on the parent level; (1) all future events, w.r.t. time t, if $o=$'ev'

$$U^{ev}: i,t,a \mapsto \{e^k_v, e^l_w, ...\}$$

with $v,w>t$ and event types k,l according to the schedule of the child contract, (2) a certain state variable x if $o=$'sv'

$$U^{sv}: i,t,x,a \mapsto \mathbb{R},$$

or (3) a particular contract attribute x of the child contract if $o=$'ca'

$$U^{ca}: i,x \mapsto y$$

with y a variable of value type of the respective attribute as per DD.

The parameters to the Child Contract Observer interface are as follows:

- i : the identifier of the child contract *observed*
- t : for $o \in \{ev, sv\}$ the time for which the respective quantity should be evaluated
- x : for $o \in \{sv, ca\}$ the quantity to be evaluated
- a : for $o \in \{ev, sv\}$ a set of contract attributes to which the evaluated quantity should be conditioned

Note that the observer interface only defines the structure of an actual observer function but not the actual implementation. Thus, the interface allows for user-defined implementations of observer functions allowing e.g. for using arbitrary data structures.

Example 12 ('ev'-Observer)**.** *The future events, w.r.t. time t, of the first leg (i.e. child contract identified by* `FirstLeg`*) of a SWAPS contract with* `CNTRL` $= PFL$ *(i.e. pay first leg) can be evaluated as* $U^{ev}(FirstLeg, t \mid \{\texttt{CNTRL} = RPL\})$.

Example 13 ('sv'-Observer)**.** *The current state, w.r.t. time t, of state variable* ***Nvl*** *of the first leg (i.e. child contract identified by* `FirstLeg`*) of a SWAPS contract with* `CNTRL`$=RFL$ *(i.e. receive first leg) can be evaluated as* $U^{sv}(FirstLeg, t, \boldsymbol{Nvl} \mid \{\texttt{CNTRL} = RPA\})$.

Example 14 ('ca'-Observer)**.** *The contract attribute* `MOC` *of the child contract Child (i.e. child contract identified by* `Child`*) of an OPTNS contract can be evaluated as* $U^{ca}(Child, \texttt{MOC})$.

For brevity we will omit the x and a function arguments wherever this does not lead to confusion.

9. Contract Types

9.1. PAM: Principal At Maturity.

PAM: Contract Schedule

Event	Schedule	Comments
AD	$\vec{t}^{AD} = (t_0, t_1, ..., t_n)$	With $t_i, i = 1, 2, ...$ a custom input
IED	$t^{IED} = \texttt{IED}$	
PR	$t^{PR} = \textbf{Tmd}_{t_0}$	
PP	$\vec{t}^{PP} = \begin{cases} \varnothing & \text{if } \texttt{PPEF} = \text{'N'} \\ (\vec{u}, \vec{v}) & \text{else} \end{cases}$ where $\vec{u} = S(s, \texttt{OPCL}, T^{MD})$ $\vec{v} = O^{rf}(\texttt{PPMO}, t)$	with $s = \begin{cases} \varnothing & \text{if } \texttt{OPANX} = \varnothing \wedge \texttt{OPCL} = \varnothing \\ \texttt{IED} + \texttt{OPCL} & \text{else if } \texttt{OPANX} = \varnothing \\ \texttt{OPANX} & \text{else} \end{cases}$
PY	$\vec{t}^{PY} = \begin{cases} \varnothing & \text{if } \texttt{PYTP} = \text{'O'} \\ \vec{t}^{PP} & \text{else} \end{cases}$	
FP	$\vec{t}^{FP} = \begin{cases} \varnothing & \text{if } \texttt{FER} = \varnothing \vee \texttt{FER} = 0 \\ S(s, \texttt{FPCL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{FPANX} = \varnothing \wedge \texttt{FPCL} = \varnothing \\ \texttt{IED} + \texttt{FPCL} & \text{else if } \texttt{FPANX} = \varnothing \\ \texttt{FPANX} & \text{else} \end{cases}$
PRD	$t^{PRD} = \texttt{PRD}$	
TD	$t^{TD} = \texttt{TD}$	
IP	$\vec{t}^{IP} = \begin{cases} \varnothing & \text{if } \texttt{IPNR} = \text{'O'} \\ S(s, \texttt{IPCL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{IPANX} = \varnothing \wedge \texttt{IPCL} = \varnothing \\ \texttt{IPCED} & \text{else if } \texttt{IPCED} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else if } \texttt{IPANX} = \varnothing \\ \texttt{IPANX} & \text{else} \end{cases}$
IPCI	$\vec{t}^{IPCI} = \begin{cases} \varnothing & \text{if } \texttt{IPCED} = \varnothing \\ S(s, \texttt{IPCL}, \texttt{IPCED}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{IPANX} = \varnothing \wedge \texttt{IPCL} = \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else if } \texttt{IPANX} = \varnothing \\ \texttt{IPANX} & \text{else} \end{cases}$
RR	$\vec{t}^{RR} = \begin{cases} \varnothing & \text{if } \texttt{RRANX} = \varnothing \wedge \texttt{RRCL} = \varnothing \\ \vec{t} \setminus t^{RRY} & \text{else if} \texttt{RRNXT} \neq \varnothing \\ \vec{t} & \text{else} \end{cases}$ where $\vec{t} = S(s, \texttt{RRCL}, T^{MD})$	with $s = \begin{cases} \texttt{IED} + \texttt{RRCL} & \text{if } \texttt{RRANX} = \varnothing \\ \texttt{RRANX} & \text{else} \end{cases}$ $t^{RRY} = \inf t \in \vec{t} \mid t > \texttt{SD}$
RRF	$t^{RRF} = \begin{cases} \varnothing & \text{if } \texttt{RRANX} = \varnothing \wedge \texttt{RRCL} = \varnothing \\ \inf t \in \vec{t} \mid t > \texttt{SD} & \text{else} \end{cases}$ where $\vec{t} = S(s, \texttt{RRCL}, T^{MD})$	with $s = \begin{cases} \texttt{IED} + \texttt{RRCL} & \text{if } \texttt{RRANX} = \varnothing \\ \texttt{RRANX} & \text{else} \end{cases}$
SC	$\vec{t}^{SC} = \begin{cases} \varnothing & \text{if } \texttt{SCEF} = \text{'000'} \\ S(s, \texttt{SCCL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{SCANX} = \varnothing \wedge \texttt{SCCL} = \varnothing \\ \texttt{IED} + \texttt{SCCL} & \text{else if } \texttt{SCANX} = \varnothing \\ \texttt{SCANX} & \text{else} \end{cases}$
CD	$t^{CD} = O^{ev}(LEICP, t_0)$	

PAM: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\textbf{Tmd}_{t_0} = \texttt{MD}$	
Nvl	$\textbf{Nvl}_{t_0} = \begin{cases} 0.0 & \text{if } \texttt{IED} > t_0 \\ R(\texttt{CNTRL}) \times \texttt{NT} & \text{else} \end{cases}$	
Nrt	$\textbf{Nrt}_{t_0} = \begin{cases} 0.0 & \text{if } \texttt{IED} > t_0 \\ \texttt{IPNR} & \text{else} \end{cases}$	

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State	Initialization per t_0	Comments
Nac	$\mathbf{Nac}_{t_0} = \begin{cases} 0.0 & \text{if} \quad \texttt{IPNR} = \varnothing \\ \texttt{IPAC} & \text{else if} \quad \texttt{IPAC} \neq \varnothing \\ Y(t^-, t_0) \times \mathbf{Nvl}_{t_0} \times \mathbf{Nrt}_{t_0} & \text{else} \end{cases}$	with $t^- = \sup t \in \vec{t}^{IP} \mid t < t_0$
Fac	$\mathbf{Fac}_{t_0} = \begin{cases} 0.0 & \text{if} \quad \texttt{FER} = \varnothing \\ \texttt{FEAC} & \text{else if} \quad \texttt{FEAC} \neq \varnothing \\ Y(t^-, t_0) \times \mathbf{Nvl}_{t_0} \times \texttt{FER} & \text{else if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t_0)}{Y(t^{FP-}, t^{FP+})} \times \texttt{FER} & \text{else} \end{cases}$	with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$, $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
Nsc	$\mathbf{Nsc}_{t_0} = \begin{cases} \texttt{SCIXSD} & \text{if} \quad \texttt{SCEF} = \text{'[x]N[x]'} \\ 1.0 & \text{else} \end{cases}$	
Isc	$\mathbf{Isc}_{t_0} = \begin{cases} \texttt{SCIXSD} & \text{if} \quad \texttt{SCEF} = \text{'I[x][x]'} \\ 1.0 & \text{else} \end{cases}$	
Prf	$\mathbf{Prf}_{t_0} = \texttt{CTS}$	
Led	$\mathbf{Led}_{t_0} = t_0$	

PAM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	0.0	$\mathbf{Nac}_{t+} = \mathbf{Nac}_{t-} + Y(\mathbf{Led}_{t-1}, t)\mathbf{Nrt}_{t-}\mathbf{Nvl}_{t-}$ $\mathbf{Led}_{t+} = t$
IED	$D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})(-1)(\texttt{NT} + \texttt{PDIED})$	$\mathbf{Nvl}_{t+} = R(\texttt{CNTRL})\texttt{NT}$ $\mathbf{Nrt}_{t+} = \begin{cases} 0.0 & \text{if} \quad \texttt{IPNR} = \varnothing \\ \texttt{IPNR} & \text{else} \end{cases}$ $\mathbf{Nac}_{t+} = \begin{cases} \texttt{IPAC} & \text{if} \quad \texttt{IPAC} \neq \varnothing \\ y\mathbf{Nvl}_{t+}\mathbf{Nrt}_{t+} & \text{if} \quad \texttt{IPANX} \neq \varnothing \wedge \texttt{IPANX} < t \\ 0.0 & \text{else} \end{cases}$ $\mathbf{Led}_{t+} = t$ with $y = Y(\texttt{IPANX}, t)$
PR	$D(\mathbf{Prf}_{t-})\mathbf{Nsc}_{t-}\mathbf{Nvl}_{t-}$	$\mathbf{Nvl}_{t+} = 0.0$ $\mathbf{Nrt}_{t+} = 0.0$ $\mathbf{Led}_{t+} = t$
PP	$D(\mathbf{Prf}_{t-})O^{rf}(\texttt{OPMO}, t)$	$\mathbf{Nac}_{t+} = \mathbf{Nac}_{t-} + Y(\mathbf{Led}_{t-}, t)\mathbf{Nrt}_{t-}\mathbf{Nvl}_{t-}$ $\mathbf{Fac}_{t+} = \begin{cases} \mathbf{Fac}_{t-} + Y(\mathbf{Led}_{t-}, t)\mathbf{Nvl}_{t-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nvl}_{t+} = \mathbf{Nvl}_{t-} - O^{rf}(\texttt{OPMO}, t)$ $\mathbf{Led}_{t+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PY	$D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})\texttt{PYRT}$ if $\texttt{PYTP} = \text{'A'}$ $c\texttt{PYRT}$ if $\texttt{PYTP} = \text{'N'}$ $c \max(0, \mathbf{Nrt}_{t-} - O^{rf}(\texttt{RRMO}, t))$ if $\texttt{PYTP} = \text{'I'}$ with $c = D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})Y(\mathbf{Led}_{t-}, t)\mathbf{Nvl}_{t-}$	$\mathbf{Nac}_{t+} = \mathbf{Nac}_{t-} + Y(\mathbf{Led}_{t-}, t)\mathbf{Nrt}_{t-}\mathbf{Nvl}_{t-}$ $\mathbf{Fac}_{t+} = \begin{cases} \mathbf{Fac}_{t-} + Y(\mathbf{Led}_{t-}, t)\mathbf{Nvl}_{t-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
FP	c if $\texttt{FEB} = \text{'A'}$ $cY(\mathbf{Led}_{t-}, t)\mathbf{Nvl}_{t-} + \mathbf{Fac}_{t-}$ if $\texttt{FEB} = \text{'N'}$ with $c = D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})\texttt{FER}$	$\mathbf{Nac}_{t+} = \mathbf{Nac}_{t-} + Y(\mathbf{Led}_{t-}, t)\mathbf{Nrt}_{t-}\mathbf{Nvl}_{t-}$ $\mathbf{Fac}_{t+} = 0.0$ $\mathbf{Led}_{t+} = t$

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Event	Payoff Function	State Transition Function
PRD	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(-1)(\texttt{PPRD} + \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-})$	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
TD	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(\texttt{PTD} + \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-})$	$\mathbf{Nvl}_{t^+} = 0.0$ $\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Fac}_{t^+} = 0.0$ $\mathbf{Nrt}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
IP	$D(\mathbf{Prf}_{t^-})\mathbf{Isc}_{t^-}(\mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-})$	$\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
IPCI	0.0	$\mathbf{Nvl}_{t^+} = \mathbf{Nvl}_{t^-} + \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\mathbf{Nrt}_{t^-}$ $\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RR	0.0	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nrt}_{t^+} = \min(\max(\mathbf{Nrt}_{t^-} + \Delta r, \texttt{RRLF}), \texttt{RRLC})$ $\mathbf{Led}_{t^+} = t$ with $\Delta r = \min(\max(O^{rf}(\texttt{RRMO}, t)\texttt{RRMT} + \texttt{RRSP} - \mathbf{Nrt}_{t^-}, \texttt{RRPF}), \texttt{RRPC})$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RRF	0.0	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nrt}_{t^+} = \texttt{RRNXT}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

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Event	Payoff Function	State Transition Function
SC	0.0	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nsc}_{t^+} = \begin{cases} \mathbf{Nsc}_{t^-} & \text{if} \quad \texttt{SCEF} = [x]0[x] \\ \frac{O^{rf}(\texttt{SCMO},t)-\texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Isc}_{t^+} = \begin{cases} \mathbf{Isc}_{t^-} & \text{if} \quad \texttt{SCEF} = 0[x][x] \\ \frac{O^{rf}(\texttt{SCMO},t)-\texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
CD	0.0	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Prf}_{t^+} = \text{'D'}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

9.2. **LAM: Linear Amortizer.**

LAM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR	$t^{PR} = S(s, \texttt{PRCL}, T^{MD})$	with $s = \begin{cases} \varnothing & \text{if} \quad \texttt{PRANX} = \varnothing \wedge \texttt{PRCL} = \varnothing \\ \texttt{IED} + \texttt{PRCL} & \text{else if} \quad \texttt{PRANX} = \varnothing \\ \texttt{PRANX} & \text{else} \end{cases}$
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP		Same as PAM
IPCI		Same as PAM
IPCB	$\vec{t}^{IPCB} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPCB} \neq \text{'NTL'} \\ S(s, \texttt{IPCBCL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if} \quad \texttt{IPCBANX} = \varnothing \wedge \texttt{IPCBCL} = \varnothing \\ \texttt{IED} + \texttt{IPCBCL} & \text{else if} \quad \texttt{IPCBANX} = \varnothing \\ \texttt{IPCBANX} & \text{else} \end{cases}$
RR		Same as PAM
RRF		Same as PAM
SC		Same as PAM
CD		Same as PAM

LAM: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0}=\begin{cases}\texttt{MD} & \text{if} \texttt{MD}\neq\varnothing\\ t^-+ceil(\frac{\texttt{NT}}{\texttt{PRNXT}})\texttt{PRCL}\end{cases}$	where $t^-=\begin{cases}\texttt{PRANX} & \text{if } \texttt{PRANX}\neq\varnothing\wedge\texttt{PRANX}\geq t_0\\ \texttt{IED}+\texttt{PRCL} & \text{else if } \texttt{IED}+\texttt{PRCL}\geq t_0\\ \sup t\in\vec{t}^{PR}\mid t<t_0 & \text{else}\end{cases}$
Nvl		Same as PAM
Nrt		Same as PAM
Nac		Same as PAM
Fac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM
Prf		Same as PAM
Led		Same as PAM
Npr	$\mathbf{Npr}_{t_0}=\begin{cases}\texttt{PRNXT} & \text{if } \texttt{PRNXT}\neq\varnothing\\ \texttt{NT}(ceil(\frac{Y(s,T^{MD})}{Y(s,s+\texttt{PRCL})}))^{-1} & \text{else}\end{cases}$	with $s=\begin{cases}\texttt{PRANX} & \text{if } \texttt{PRANX}\neq\varnothing\wedge\texttt{PRANX}>t_0\\ \texttt{IED}+\texttt{PRCL} & \text{else if } \texttt{PRANX}=\varnothing\wedge\texttt{IED}+\texttt{PRCL}>t_0\\ t^- & \text{else}\end{cases}$ and where $t^-=\sup t\in\vec{t}^{PR}\mid t<t_0$
Icb	$\mathbf{Icb}_{t_0}=\begin{cases}0.0 & \text{if } t_0<\texttt{IED}\\ R(\texttt{CNTRL})\texttt{NT} & \text{else if } \texttt{IPCB}=\text{'NT'}\\ R(\texttt{CNTRL})\texttt{IPCBA} & \text{else}\end{cases}$	

LAM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	$\mathbf{Nvl}_{t^+}=R(\texttt{CNTRL})\texttt{NT}$ $\mathbf{Nrt}_{t^+}=\texttt{IPNR}$ $\mathbf{Nac}_{t^+}=\begin{cases}\texttt{IPAC} & \text{if } \texttt{IPAC}\neq\varnothing\\ y\mathbf{Nvl}_{t^+}\mathbf{Nrt}_{t^+} & \text{if } \texttt{IPANX}\neq\varnothing\wedge\texttt{IPANX}<t\\ 0.0 & \text{else}\end{cases}$ $\mathbf{Led}_{t^+}=t$ $\mathbf{Icb}_{t^+}=\begin{cases}R(\texttt{CNTRL})\texttt{NT} & \text{if } \texttt{IPCB}=\text{'NT'}\\ R(\texttt{CNTRL})\texttt{IPCBA} & \text{else}\end{cases}$
PR	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})\mathbf{Nsc}_{t^-}\mathbf{Npr}_{t^-}$	$\mathbf{Nvl}_{t^+}=\mathbf{Nvl}_{t^-}-R(\texttt{CNTRL})\mathbf{Npr}_{t^-}$ $\mathbf{Fac}_{t^+}=\begin{cases}\mathbf{Fac}_{t^-}+Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB}=\text{'N'}\\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else}\end{cases}$ $\mathbf{Icb}_{t^+}=\begin{cases}\mathbf{Icb}_{t^-} & \text{if } \texttt{IPCB}\neq\text{'NT'}\\ \mathbf{Nvl}_{t^+} & \text{else}\end{cases}$ $\mathbf{Led}_{t^+}=t$ with $t^{FP-}=\sup t\in\vec{t}^{FP}\mid t<t_0$ $t^{FP+}=\inf t\in\vec{t}^{FP}\mid t>t_0$

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Event	Payoff Function	State Transition Function
PP	POF_PP_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nvl}_{t^+} = \mathbf{Nvl}_{t^-} - O^{rf}(\texttt{OPMO},t)$ $\mathbf{Icb}_{t^+} = \begin{cases} \mathbf{Icb}_{t^-} & \text{if } \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nvl}_{t^+} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PY	POF_PY_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
FP	POF_FP_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
PRD	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(-1)(\texttt{PPRD} + \mathbf{Nac}_{t^-} +$ $Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-})$	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
TD	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(\texttt{PTD} + \mathbf{Nac}_{t^-} +$ $Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-})$	STF_TD_PAM()
IP	$D(\mathbf{Prf}_{t^-})\mathbf{Isc}_{t^-}(\mathbf{Nac}_{t^-} +$ $Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-})$	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	$\mathbf{Nvl}_{t^+} = \mathbf{Nvl}_{t^-} + \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Icb}_{t^+} = \begin{cases} \mathbf{Icb}_{t^-} & \text{if } \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nvl}_{t^+} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
IPCB	0.0	$\mathbf{Icb}_{t^+} = \mathbf{Nvl}_{t^-}$ $\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

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Event	Payoff Function	State Transition Function
RR	POF_RR_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nrt}_{t^+} = \min(\max(\mathbf{Nrt}_{t^-} + \Delta r, \texttt{RRLF}), \texttt{RRLC})$ $\mathbf{Led}_{t^+} = t$ with $\Delta r = \min(\max(O^{rf}(\texttt{RRMO}, t)\texttt{RRMT} + \texttt{RRSP} - \mathbf{Nrt}_{t^-}, \texttt{RRPF}), \texttt{RRPC})$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RRF	POF_RRF_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nrt}_{t^+} = \texttt{RRNXT}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
SC	POF_SC_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nsc}_{t^+} = \begin{cases} \mathbf{Nsc}_{t^-} & \text{if} \quad \texttt{SCEF} = [x]0[x] \\ \frac{O^{rf}(\texttt{SCMO}, t) - \texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Isc}_{t^+} = \begin{cases} \mathbf{Isc}_{t^-} & \text{if} \quad \texttt{SCEF} = 0[x][x] \\ \frac{O^{rf}(\texttt{SCMO}, t) - \texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
CD	POF_CD_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Prf}_{t^+} = \text{'D'}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

9.3. **LAX: Exotic Linear Amortizer.**

LAX: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR	$\vec{t}^{PR} = \begin{cases} \{t_1, t_2, ..., t_i, ...\} & \text{if} \quad \texttt{ARPRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else} \end{cases}$ with $s_i = S(\texttt{ARPRANX}_i, \vec{C}_i, \texttt{ARPRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARINCDEC} \mid\} \mid \texttt{ARINCDEC}_i = \text{'DEC'}$	with $\vec{C} = \begin{cases} \texttt{ARPRCL} & \text{if} \quad \mid \texttt{ARPRCL} \mid = \mid \texttt{ARPRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else} \end{cases}$ where $n = \mid \texttt{ARPRANX} \mid, c_k = \texttt{ARPRCL}_1 \forall k$

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Event	Schedule	Comments
PI	$\vec{t}^{PI} = \begin{cases}\{t_1, t_2, ..., t_i, ...\} & \text{if} \quad \texttt{ARPRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else}\end{cases}$ with $s_i = S(\texttt{ARPRANX}_i, \vec{C}_i, \texttt{ARPRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARINCDEC} \mid\} \mid \texttt{ARINCDEC}_i = \text{'INC'}$	with $\vec{C} = \begin{cases}\texttt{ARPRCL} & \text{if} \quad \mid \texttt{ARPRCL} \mid = \mid \texttt{ARPRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else}\end{cases}$ where $n = \mid \texttt{ARPRANX} \mid, c_k = \texttt{ARPRCL}_1 \forall k$
PRF	$\vec{t}^{PRF} = \texttt{ARPRANX}$	
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP	$\vec{t}^{IP} = S(\texttt{ARIPANX}, \texttt{ARIPCL}, \mathbf{Tmd}_{t_0})$	
IPCI		Same as PAM
IPCB		Same as LAM
RR	$\vec{t}^{RR} = \begin{cases}\{t_1, t_2, ..., t_i, ...\} & \text{if} \quad \texttt{ARRRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else}\end{cases}$ with $s_i = S(\texttt{ARRRANX}_i, \vec{C}_i, \texttt{ARRRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARFIXVAR} \mid\} \mid \texttt{ARFIXVAR}_i = \text{'V'}$	with $\vec{C} = \begin{cases}\texttt{ARRRCL} & \text{if} \quad \mid \texttt{ARRRCL} \mid = \mid \texttt{ARRRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else}\end{cases}$ where $n = \mid \texttt{ARRRANX} \mid, c_k = \texttt{ARRRCL}_1 \forall k$
RRF	$\vec{t}^{RRF} = \begin{cases}\{t_1, t_2, ..., t_i, ...\} & \text{if} \quad \texttt{ARRRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else}\end{cases}$ with $s_i = S(\texttt{ARRRANX}_i, \vec{C}_i, \texttt{ARRRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARFIXVAR} \mid\} \mid \texttt{ARFIXVAR}_i = \text{'F'}$	with $\vec{C} = \begin{cases}\texttt{ARRRCL} & \text{if} \quad \mid \texttt{ARRRCL} \mid = \mid \texttt{ARRRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else}\end{cases}$ where $n = \mid \texttt{ARRRANX} \mid, c_k = \texttt{ARRRCL}_1 \forall k$
SC		Same as PAM
CD		Same as PAM

LAX: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \begin{cases}\texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ \inf t > t_0 \mid N(t) = 0 & \text{else}\end{cases}$ with $N(t) = \texttt{NT} + \sum_{i=1}^{n(t)} (-1)^k \texttt{ARPRNXT}_i \mid s_i \mid$	where $n(t) = \begin{cases}\sup k \in \mathbb{N} \mid \texttt{ARPRANX}_k < t & \text{if} \quad t < max(\texttt{ARPRANX}) \\ \mid \texttt{ARPRANX} \mid & \text{else}\end{cases}$ $k = \begin{cases}0 & \text{if} \quad \texttt{ARINCDEC}_i = \text{'INC'} \\ 1 & \text{else}\end{cases}$ $s_i = \begin{cases}\{\texttt{ARPRANX}_i\} & \text{if} \quad \texttt{ARPRCL} = \varnothing \\ S(\texttt{ARPRANX}_i, \vec{C}_i, T_i) & \text{else}\end{cases}$ $T_i = \begin{cases}\texttt{ARPRANX}_{i+1} & \text{if} \quad i < \mid \texttt{ARPRANX} \mid \\ t & \text{else}\end{cases}$ $\vec{C} = \begin{cases}\texttt{ARRRCL} & \text{if} \quad \mid \texttt{ARRRCL} \mid = \mid \texttt{ARRRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else}\end{cases}$
Nvl		Same as PAM
Nrt		Same as PAM
Nac		Same as PAM
Fac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM
Prf		Same as PAM
Led		Same as PAM
Npr	$\mathbf{Npr}_{t_0} = \begin{cases}0.0 & \text{if} \quad t_0 \geq \texttt{ARPRANX}_1 \\ \texttt{ARPRNXT}_i & \text{else}\end{cases}$	where $i = \sup k \in \mathbb{N} \mid \texttt{ARPRANX}_k < t_0$

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State	Initialization per t_0	Comments
Icb		Same as LAM

LAX: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	STF_IED_LAM()
PR	POF_PR_LAM()	STF_PR_LAM()
PI	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(-1)\mathbf{Nsc}_{t^-}\mathbf{Npr}_{t^-}$	$\mathbf{Nvl}_{t^+}=\mathbf{Nvl}_{t^-}+R(\texttt{CNTRL})\mathbf{Npr}_{t^-}$ $\mathbf{Nac}_{t^+}=\mathbf{Nac}_{t^-}+Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+}=\begin{cases}\mathbf{Fac}_{t^-}+Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB}=\text{'N'}\\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else}\end{cases}$ $\mathbf{Icb}_{t^+}=\begin{cases}\mathbf{Icb}_{t^-} & \text{if } \texttt{IPCB}\neq\text{'NT'}\\ \mathbf{Nvl}_{t^+} & \text{else}\end{cases}$ $\mathbf{Led}_{t^+}=t$ with $t^{FP-}=\sup t\in\vec{t}^{FP}\mid t<t_0$ $t^{FP+}=\inf t\in\vec{t}^{FP}\mid t>t_0$
PRF	0.0	$\mathbf{Npr}_{t^+}=\texttt{ARPRNXT}_i$ $\mathbf{Nac}_{t^+}=\mathbf{Nac}_{t^-}+Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+}=\begin{cases}\mathbf{Fac}_{t^-}+Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB}=\text{'N'}\\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else}\end{cases}$ $\mathbf{Icb}_{t^+}=\begin{cases}\mathbf{Icb}_{t^-} & \text{if } \texttt{IPCB}\neq\text{'NT'}\\ \mathbf{Nvl}_{t^+} & \text{else}\end{cases}$ $\mathbf{Led}_{t^+}=t$ with $i=\sup k\in\mathbb{N}\mid\texttt{ARPRANX}_k=t$ $t^{FP-}=\sup t\in\vec{t}^{FP}\mid t<t_0$ $t^{FP+}=\inf t\in\vec{t}^{FP}\mid t>t_0$
PP	POF_PP_PAM()	STF_PP_LAM()
PY	POF_PY_PAM()	STF_PY_LAM()
FP	POF_FP_PAM()	STF_FP_LAM()
PRD	POF_PRD_LAM()	STF_PRD_LAM()
TD	POF_TD_LAM()	STF_TD_PAM()
IP	POF_IP_LAM()	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	POF_RR_PAM()	$\mathbf{Nac}_{t^+}=\mathbf{Nac}_{t^-}+Y(\mathbf{Led}_{t^-},t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+}=\begin{cases}\mathbf{Fac}_{t^-}+Y(\mathbf{Led}_{t^-},t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB}=\text{'N'}\\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else}\end{cases}$ $\mathbf{Nrt}_{t^+}=\min(\max(\mathbf{Nrt}_{t^-}+\Delta r,\texttt{RRLF}),\texttt{RRLC})$ $\mathbf{Led}_{t^+}=t$ with $\Delta r=\min(\max(O^{rf}(\texttt{RRMO},t)\texttt{RRMT}+\texttt{ARRATE}_i-\mathbf{Nrt}_{t^-},\texttt{RRPF}),\texttt{RRPC})$ $i=\sup k\in\mathbb{N}\mid\texttt{ARPRANX}_k=t$ $t^{FP-}=\sup t\in\vec{t}^{FP}\mid t<t_0$ $t^{FP+}=\inf t\in\vec{t}^{FP}\mid t>t_0$

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Event	Payoff Function	State Transition Function
RRF	POF_RRF_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nrt}_{t^+} = \texttt{ARRATE}_i$ $\mathbf{Led}_{t^+} = t$ with $i = \sup k \in \mathbb{N} \mid \texttt{ARPRANX}_k = t$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
SC	POF_SC_PAM()	STF_SC_LAM()
CD	POF_CD_PAM()	STF_CD_LAM()

9.4. **NAM: Negative Amortizer.**

NAM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR		Same as LAM
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP	$\vec{t}^{IP} = (\vec{u}, \vec{v})$ where $\vec{u} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPANX} = \varnothing \wedge \texttt{IPCL} = \varnothing \\ \varnothing & \text{if} \quad \texttt{IPCED} \neq \varnothing \wedge \texttt{IPCED} \geq \texttt{T} \\ S(r, \texttt{IPCL}, T) & \text{else} \end{cases}$ $\vec{v} = S(s, \texttt{PRCL}, T^{MD})$	with $r = \begin{cases} \texttt{IPCED} & \text{if} \quad \texttt{IPCED} \neq \varnothing \\ \texttt{IPANX} & \text{else if} \quad \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else if} \quad \texttt{IPCL} \neq \varnothing \\ \varnothing & \text{else} \end{cases}$ $T = s - \texttt{PRCL}$ $s = \begin{cases} \texttt{IED} + \texttt{PRCL} & \text{if} \quad \texttt{PRANX} = \varnothing \\ \texttt{PRANX} & \text{else} \end{cases}$
IPCI		Same as PAM
IPCB		Same as LAM
RR		Same as PAM
RRF		Same as PAM
SC		Same as PAM
CD		Same as PAM

NAM: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ t^- + n\texttt{PRCL} & \text{else} \end{cases}$ with $n = ceil(\frac{\texttt{NT}}{\texttt{PRNXT} - \texttt{NT}Y(t^-, t^- + \texttt{PRCL})\texttt{IPNR}})$	where $t^- = \begin{cases} \texttt{PRANX} & \text{if} \quad \texttt{PRANX} \neq \varnothing \wedge \texttt{PRANX} \geq t_0 \\ \texttt{IED} + \texttt{PRCL} & \text{else if} \quad \texttt{IED} + \texttt{PRCL} \geq t_0 \\ \sup t \in t^{PR} \mid t < t_0 & \text{else} \end{cases}$
Nvl		Same as PAM
Nrt		Same as PAM
Nac		Same as PAM
Fac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM
Prf		Same as PAM
Led		Same as PAM

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State	Initialization per t_0	Comments
Npr	$\mathbf{Npr}_{t_0} = R(\texttt{CNTRL})\texttt{PRNXT}$	
Icb		Same as LAM

NAM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	STF_IED_LAM()
PR	$D(\mathbf{Prf}_{t^-})\mathbf{Nsc}_{t^-}(\mathbf{Npr}_{t^-} - \mathbf{Nac}_{t^-} - Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-})$	$\mathbf{Nvl}_{t^+} = \mathbf{Nvl}_{t^-} - (\mathbf{Npr}_{t^-} - \mathbf{Nac}_{t^+})$ $\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Icb}_{t^+} = \begin{cases} \mathbf{Icb}_{t^-} & \text{if } \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nvl}_{t^+} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PP	POF_PP_PAM()	STF_PP_LAM()
PY	POF_PY_PAM()	STF_PY_LAM()
FP	POF_FP_PAM()	STF_FP_LAM()
PRD	POF_PRD_LAM()	STF_PRD_LAM()
TD	POF_TD_LAM()	STF_TD_PAM()
IP	POF_IP_LAM()	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	POF_RR_PAM()	STF_RR_LAM()
RRF	POF_RRF_PAM()	STF_RRF_LAM()
SC	POF_SC_PAM()	STF_SC_LAM()
CD	POF_CD_PAM()	STF_CD_LAM()

9.5. **ANN: Annuity.**

ANN: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR		Same as LAM
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP		Same as NAM
IPCI		Same as PAM
IPCB		Same as LAM
RR		Same as PAM
RRF		Same as PAM
SC		Same as PAM
CD		Same as PAM

ANN: State Variables Initialization

State	Initialization per t_0	Comments
Tmd		Same as NAM
Nvl		Same as PAM

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State	Initialization per t_0	Comments
Nrt		Same as PAM
Nac		Same as PAM
Fac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM
Prf		Same as PAM
Led		Same as PAM
Npr	$\mathbf{Npr}_{t_0} = \begin{cases} R(\mathtt{CNTRL})\mathtt{PRNXT} & \text{if} \quad \mathtt{PRNXT} \neq \varnothing \\ (\mathtt{NT} + \mathbf{Nac}_{t_0})\frac{todo}{todo} & \text{else} \end{cases}$	where $n = \lvert\vec{t}\rvert$ with $\lvert a\rvert$ indicating the cardinality of set a
Icb		Same as LAM

ANN: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	STF_IED_LAM()
PR	POF_PR_NAM()	STF_PR_NAM()
PP	POF_PP_PAM()	STF_PP_LAM()
PY	POF_PY_PAM()	STF_PY_LAM()
FP	POF_FP_PAM()	STF_FP_LAM()
PRD	POF_PRD_LAM()	STF_PRD_LAM()
TD	POF_TD_LAM()	STF_TD_PAM()
IP	POF_IP_LAM()	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	POF_RR_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\mathtt{FER} & \text{if} \quad \mathtt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\mathtt{FER} & \text{else} \end{cases}$ $\mathbf{Nrt}_{t^+} = \min(\max(\mathbf{Nrt}_{t^-} + \Delta r, \mathtt{RRLF}), \mathtt{RRLC})$ $\mathbf{Npr}_{t^+} = A(t, \mathbf{Tmd}_{t^+}, \mathbf{Nvl}_{t^+}, \mathbf{Nac}_{t^+}, \mathbf{Nrt}_{t^+})$ $\mathbf{Led}_{t^+} = t$ with $\Delta r = \min(\max(O^{rf}(\mathtt{RRMO}, t)\mathtt{RRMT} + \mathtt{RRSP} - \mathbf{Nrt}_{t^-}, \mathtt{RRPF}), \mathtt{RRPC})$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RRF	POF_RRF_PAM()	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Icb}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\mathtt{FER} & \text{if} \quad \mathtt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})}\mathtt{FER} & \text{else} \end{cases}$ $\mathbf{Nrt}_{t^+} = \mathtt{RRNXT}$ $\mathbf{Npr}_{t^+} = A(t, \mathbf{Tmd}_{t^+}, \mathbf{Nvl}_{t^+}, \mathbf{Nac}_{t^+}, \mathbf{Nrt}_{t^+})$ $\mathbf{Led}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
SC	POF_SC_PAM()	STF_SC_LAM()
CD	POF_CD_PAM()	STF_CD_LAM()

9.6. **CLM: Call Money.**

CLM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR		Same as PAM

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Event	Schedule	Comments
FP		Same as PAM
IP	$t^{IP} = \mathbf{Tmd}_{t_0}$	
IPCI	$\vec{t}^{IPCI} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPNR} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Tmd}_{t_0}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{IPANX} & \text{if} \quad \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else} \end{cases}$
RR		Same as PAM
RRF		Same as PAM
CD		Same as PAM

CLM: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ s & \text{else if} \quad O^{ev}(\texttt{CID}, t_0) \neq \{\} \\ t^{max} & \text{else} \end{cases}$	where $s = \sup t \in \tau(O^{ev}(\texttt{CID}, t_0))$
Nvl		Same as PAM
Nrt		Same as PAM
Nac		Same as PAM
Fac		Same as PAM
Prf		Same as PAM
Led		Same as PAM

CLM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(-1)\texttt{NT}$	STF_IED_PAM()
PR	POF_PR_PAM()	STF_PR_PAM()
FP	POF_FP_PAM()	STF_FP_PAM()
IP	$D(\mathbf{Prf}_{t^-})(\mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-})$	$\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
IPCI	POF_IPCI_PAM()	STF_IPCI_PAM()
RR	POF_RR_PAM()	STF_RR_PAM()
RRF	POF_RRF_PAM()	STF_RRF_PAM()
CD	POF_CD_PAM()	STF_CD_PAM()

9.7. **UMP: Undefined Maturity Profile.**

UMP: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR	$\vec{t}^{PR} = O^{ev}(\texttt{CID}, t_0)$	
FP		Same as PAM
IPCI	$\vec{t}^{IPCI} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPNR} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Tmd}_{t_0}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{IPANX} & \text{if} \quad \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else} \end{cases}$
RR		Same as PAM
RRF		Same as PAM
CD		Same as PAM

UMP: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \begin{cases} s & \text{if} \quad O^{ev}(\texttt{CID}, t_0) \neq \{\} \\ t^{max} & \text{else} \end{cases}$	where $s = \sup t, t \in \tau(O^{ev}(\texttt{CID}, t_0))$
Nvl		Same as PAM
Nrt		Same as PAM
Nac		Same as PAM
Fac		Same as PAM
Prf		Same as PAM
Led		Same as PAM

UMP: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_CLM()	STF_IED_PAM()
PR	$f(e_t^{PR})$	$\mathbf{Nac}_{t^+} = \mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^-,t)}{Y(t^-,t^+)}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nvl}_{t^+} = \mathbf{Nvl}_{t^-} - f(e_t^{PR})$ $\mathbf{Led}_{t^+} = t$
FP	POF_FP_PAM()	STF_FP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_PAM()
RR	POF_RR_PAM()	STF_RR_PAM()
RRF	POF_RRF_PAM()	STF_RRF_PAM()
CD	POF_CD_PAM()	STF_CD_PAM()

9.8. CSH: Cash.

CSH: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM

CSH: State Variables Initialization

State	Initialization per t_0	Comments
Nvl	$\mathbf{Nvl}_{t_0} = R(\texttt{CNTRL})\texttt{NT}$	
Led		Same as PAM

CSH: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Led}_{t^+} = t$

9.9. STK: Stock.

STK: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
$\text{DV}^{(fix)}$	$t^{DV^{(fix)}} = \begin{cases} \varnothing & \text{if} \quad \texttt{DVNP} = \varnothing \\ \texttt{DVANX} & \text{else} \end{cases}$	
DV	$\vec{t}^{DV} = \begin{cases} \varnothing & \text{if} \quad \texttt{DVANX} = \varnothing \wedge \texttt{DVCL} = \varnothing \\ S(s, \texttt{DVCL}, t^{max}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{DVANX} & \text{if} \quad \texttt{DVNP} = \varnothing \\ \texttt{DVANX} + \texttt{DVCL} & \text{else} \end{cases}$
CD		Same as PAM

STK: State Variables Initialization

State	Initialization per t_0	Comments
Prf		Same as PAM
Led		Same as PAM

STK: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Led}_{t+} = t$
PRD	$D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})(-1)\texttt{PPRD}$	$\mathbf{Led}_{t+} = t$
TD	$D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})\texttt{PTD}$	$\mathbf{Led}_{t+} = t$
$\text{DV}^{(fix)}$	$D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})\texttt{DVNP}$	$\mathbf{Led}_{t+} = t$
DV	$D(\mathbf{Prf}_{t-})R(\texttt{CNTRL})O^{rf}(\texttt{DVMO}, t)$	$\mathbf{Led}_{t+} = t$
CD	POF_CD_PAM()	$\mathbf{Prf}_{t+} = O^{rf}(\texttt{LEICP}, t)$ $\mathbf{Led}_{t+} = t$

9.10. COM: Commodity.

COM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as STK
TD		Same as STK

COM: State Variables Initialization

State	Initialization per t_0	Comments
Led		Same as STK

COM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_STK()
PRD	POF_PRD_STK()	STF_PRD_STK()
TD	POF_PRD_STK()	STF_PRD_STK()

9.11. FXOUT: Foreign Exchange Outright.

FXOUT: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
STD	$t^{STD} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'D'} \\ \mathbf{Tmd}_{t_0} & \text{else} \end{cases}$	
$\text{STD}^{(1)}$	$t^{STD} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'S'} \\ \mathbf{Tmd}_{t_0} & \text{else} \end{cases}$	
$\text{STD}^{(2)}$	$t^{STD} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'S'} \\ \mathbf{Tmd}_{t_0} & \text{else} \end{cases}$	
CD		Same as PAM

FXOUT: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \begin{cases} \texttt{MD} & \text{if } \texttt{STD} = \varnothing \\ \texttt{STD} & \text{else} \end{cases}$	

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State	Initialization per t_0	Comments
Prf		Same as PAM
Led		Same as PAM

FXOUT: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_STK()
PRD	POF_PRD_STK()	STF_PRD_STK()
TD	POF_TD_STK()	STF_TD_STK()
STD	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(\texttt{NT} - O^{rf}(i, \mathbf{Tmd}_t)\texttt{NT2})$ where $i = concat(\texttt{CUR2},\text{"/"},\texttt{CUR})$ and $concat(x, y, z)$ indicates the string concatenation function	$\mathbf{Led}_{t^+} = t$
$\text{STD}^{(1)}$	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})\texttt{NT}$	$\mathbf{Led}_{t^+} = t$
$\text{STD}^{(2)}$	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(-1)\texttt{NT2}$	$\mathbf{Led}_{t^+} = t$
CD	POF_CD_PAM()	STF_CD_STK()

9.12. **SWPPV: Plain Vanilla Interest Rate Swap.**

SWPPV: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IED		Same as PAM
PR		Same as PAM
IP	$\vec{t}^{IP} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'D'} \\ \mathbf{Tmd}_{t_0} & \text{else if} \texttt{IPCL} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Tmd}_{t_0}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{IPANX} & \text{if } \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else} \end{cases}$
$\text{IP}^{(fix)}$	$\vec{t}^{IP^{(fix)}} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'S'} \\ \mathbf{Tmd}_{t_0} & \text{else if} \texttt{IPCL} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Tmd}_{t_0}) & \text{else} \end{cases}$	
$\text{IP}^{(var)}$	$\vec{t}^{IP^{(var)}} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'S'} \\ \mathbf{Tmd}_{t_0} & \text{else if} \texttt{IPCL} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Tmd}_{t_0}) & \text{else} \end{cases}$	
RR	$\vec{t}^{RR} = S(s, \texttt{RRCL}, \mathbf{Tmd}_{t_0})$	where $s = \begin{cases} \texttt{RRANX} & \text{if } \texttt{RRANX} \neq \varnothing \\ \texttt{IED} + \texttt{RRCL} & \text{else} \end{cases}$
CD		Same as PAM

SWPPV: State Variables Initialization

State	Initialization per t_0	Comments
Tmd		Same as PAM
Nvl		Same as PAM
Nrt	$\mathbf{Nrt}_{t_0} = \begin{cases} 0.0 & \text{if } \texttt{IED} > t_0 \\ \texttt{IPNR2} & \text{else} \end{cases}$	
Nac	$\mathbf{Nac}_{t_0} = \begin{cases} \texttt{IPAC} & \text{if } \texttt{IPAC} \neq \varnothing \\ Y(t^-, t_0)\mathbf{Nvl}_{t_0}(\texttt{IPNR} - \mathbf{Nrt}_{t_0}) & \text{else} \end{cases}$	with $t^- = \sup t, t \in t^{IP}, t < t_0$
Nac1	$\mathbf{Nac1}_{t_0} = Y(t^-, t_0)\mathbf{Nvl}_{t_0}\texttt{IPNR}$	with $t^- = \sup t, t \in t^{IP}, t < t_0$
Nac2	$\mathbf{Nac2}_{t_0} = Y(t^-, t_0)\mathbf{Nvl}_{t_0}\mathbf{Nrt}_{t_0}$	with $t^- = \sup t, t \in t^{IP}, t < t_0$
Prf		Same as PAM
Led		Same as PAM

SWPPV: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Nac}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}(\texttt{IPNR} - \mathbf{Nrt}_{t_0})$ $\mathbf{Nac1}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\texttt{IPNR}$ $\mathbf{Nac2}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\mathbf{Nrt}_{t_0}$ $\mathbf{Led}_{t^+} = t$
IED	0.0	$\mathbf{Nvl}_{t^+} = R(\texttt{CNTRL})\texttt{NT}$ $\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Nac1}_{t^+} = 0.0$ $\mathbf{Nac2}_{t^+} = 0.0$ $\mathbf{Nrt}_{t^+} = \texttt{IPNR2}$ $\mathbf{Led}_{t^+} = t$
PR	0.0	$\mathbf{Nvl}_{t^+} = 0.0$ $\mathbf{Nrt}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
PRD	POF_PRD_STK()	$\mathbf{Nac}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}(\texttt{IPNR} - \mathbf{Nrt}_{t_0})$ $\mathbf{Nac1}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\texttt{IPNR}$ $\mathbf{Nac2}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\mathbf{Nrt}_{t_0}$ $\mathbf{Led}_{t^+} = t$
TD	POF_TD_STK()	$\mathbf{Nvl}_{t^+} = 0.0$ $\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Nac1}_{t^+} = 0.0$ $\mathbf{Nac2}_{t^+} = 0.0$ $\mathbf{Nrt}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
IP	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(\mathbf{Nac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)(\texttt{IPNR} - \mathbf{Nrt}_{t^-})\mathbf{Nvl}_{t^-})$	$\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
$\text{IP}^{(fix)}$	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(\mathbf{Nac1}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\texttt{IPNR}\mathbf{Nvl}_{t^-})$	$\mathbf{Nac1}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
$\text{IP}^{(var)}$	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})(\mathbf{Nac2}_{t^-} - Y(\mathbf{Led}_{t^-}, t)\mathbf{Nrt}_{t^-}\mathbf{Nvl}_{t^-})$	$\mathbf{Nac2}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
RR	POF_RR_PAM()	$\mathbf{Nac}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}(\texttt{IPNR} - \mathbf{Nrt}_{t_0})$ $\mathbf{Nac1}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\texttt{IPNR}$ $\mathbf{Nac2}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\mathbf{Nrt}_{t_0}$ $\mathbf{Nrt}_{t^+} = \texttt{RRMLT}O^{rf}(\texttt{RRMO}, t) + \texttt{RRSP}$ $\mathbf{Led}_{t^+} = t$
CD	POF_CD_PAM()	$\mathbf{Nac}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}(\texttt{IPNR} - \mathbf{Nrt}_{t_0})$ $\mathbf{Nac1}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\texttt{IPNR}$ $\mathbf{Nac2}_{t^+} = Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t_0}\mathbf{Nrt}_{t_0}$ $\mathbf{Prf}_{t^+} = \text{'D'}$ $\mathbf{Led}_{t^+} = t$

9.13. **SWAPS: Swap.**

SWAPS: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM

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Event	Schedule	Comments
k	$\{e^k_t\} = \begin{cases}\{e^{k,1}_t\} \cup \{e^{l,2}_s\} & \text{if } \texttt{DS} = \text{'D'} \\ \{e^{k,1}_t\} + \{e^{l,2}_s\} & \text{else}\end{cases}$ with $\{e^{k,1}_t\} + \{e^{l,2}_s\} = U \cup V$ and $U = \{e^{k,1}_t\} \triangle \{e^{l,2}_s\}$ $V = \{x^m_\tau + y^m_\tau\}$ where for any two events $x^k_t \in \{e^{k,1}_t\}$, $y^l_s \in \{e^{l,2}_s\}$ we have $x^k_t = y^l_s \iff t = s \wedge k = l$, $\triangle$ is the *distinct union*-operator, and $x^k_t + y^l_s = z^m_\tau$ with $\tau = t = s$, $m = k = l \in$ {IED,IP,PR} indicates that any two congruent events of type IED, IP, or PR are *merged* into a new *aggregate* event (see payoff and state transition function below).	with $\{e^{k,1}_t\} = U^{ev}(FirstLeg, t_0 \mid \{\texttt{CNTRL} = r^{(1)}\})$ $\{e^{l,2}_s\} = U^{ev}(SecondLeg, t_0 \mid \{\texttt{CNTRL} = r^{(2)}\})$ $r^{(1)} = \begin{cases} RPA & \text{if } \texttt{CNTRL} = RFL \\ RPL & \text{else}\end{cases}$ $r^{(2)} = \begin{cases} RPL & \text{if } \texttt{CNTRL} = RFL \\ RPA & \text{else}\end{cases}$

SWAPS: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \max(U^{sv}(FirstLeg, t_0, \mathbf{Tmd}), U^{sv}(SecondLeg, t_0, \mathbf{Tmd}))$	
Nac	$\mathbf{Nac}_{t_0} = U^{sv}(FirstLeg, t_0, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(SecondLeg, t_0, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(2)}\})$	with $r^{(1)} = \begin{cases} RPA & \text{if } \texttt{CNTRL} = RFL \\ RPL & \text{else}\end{cases}$ $r^{(2)} = \begin{cases} RPL & \text{if } \texttt{CNTRL} = RFL \\ RPA & \text{else}\end{cases}$
Prf		Same as PAM
Led		Same as PAM

SWAPS: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Nac}_{t^+} = U^{sv}(FirstLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(SecondLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(2)}\})$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^-,t)}{Y(t^-,t^+)}\texttt{FER} & \text{else}\end{cases}$ $\mathbf{Led}_{t^+} = t$
PRD	$D(\mathbf{Prf}_{t^-})((-1)\texttt{PPRD} + U^{sv}(FirstLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(SecondLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(2)}\}))$	$\mathbf{Nac}_{t^+} = U^{sv}(FirstLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(SecondLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(2)}\})$ $\mathbf{Led}_{t^+} = t$
TD	$D(\mathbf{Prf}_{t^-})(\texttt{PTD} + U^{sv}(FirstLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(SecondLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(2)}\}))$	$\mathbf{Nac}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
z^m_τ	$f(x^m_\tau) + f(y^m_\tau)$	$\mathbf{Nac}_{t^+} = U^{sv}(FirstLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(SecondLeg, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(2)}\})$ $\mathbf{Led}_{t^+} = t$

9.14. CAPFL: Cap-Floor.

CAPFL: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM

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Continued from previous page

Event	Schedule	Comments
k	$\{e_t^k\} = \{x_t^k + y_s^l\}$ for all events $x_t^k \in U^{ev}(Child, t_0 \mid \{\texttt{CNTRL} = RPA\})$, $y_s^l \in U^{ev}(Child, t_0 \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\})$ with $t = s \wedge k = l = IP$. That is, any two congruent events of the child-contract schedule, evaluated once without RRLC, RRLF defined and once with the attributes defined, which are of type IP are *merged* into a new *aggregate* event (see payoff and state transition function below).	

CAPFL: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \max(U^{sv}(Child, t_0, \mathbf{Tmd} \mid \{\texttt{CNTRL} = RPA\}), U^{sv}(Child, t_0, \mathbf{Tmd} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$	
Nac	$\mathbf{Nac}_{t_0} = R(\texttt{CNTRL})abs(U^{sv}(Child, t_0, \mathbf{Nac} \mid \{\texttt{CNTRL} = r^{(1)}\}) - U^{sv}(Child, t_0, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$	
Prf		Same as PAM
Led		Same as PAM

CAPFL: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Nac}_{t+} = R(\texttt{CNTRL})abs(U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$ $\mathbf{Led}_{t+} = t$
PRD	$D(\mathbf{Prf}_{t-})((-1)\texttt{PPRD} + R(\texttt{CNTRL})abs(U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\})))$	$\mathbf{Nac}_{t+} = R(\texttt{CNTRL})abs(U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$ $\mathbf{Led}_{t+} = t$
TD	$D(\mathbf{Prf}_{t-})(\texttt{PTD} + \mathrm{R}(\texttt{CNTRL})\mathrm{abs}(U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(Child, t, \mathbf{Nac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\})))$	$\mathbf{Nac}_{t+} = 0.0$ $\mathbf{Led}_{t+} = t$
z_τ^m	$R(\texttt{CNTRL})abs(f(x_\tau^m) - f(y_\tau^m))$ where $abs(u)$ defines that the absolute value of u is taken.	$\mathbf{Nac}_{t+} = 0.0$ $\mathbf{Led}_{t+} = t$

9.15. **OPTNS: Option.**

OPTNS: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
XD	$t^{XD} = \texttt{MD}$	
STD	$t^{STD} = \begin{cases} \texttt{MD} & \text{if } \texttt{STD} = \varnothing \\ \texttt{STD} & \text{else} \end{cases}$	
CD		Same as PAM

OPTNS: State Variables Initialization

State	Initialization per t_0	Comments
Pos	$\mathbf{Pos}_{t_0} = \begin{cases} 0.0 & \text{if} \quad t_0 \leq \texttt{MD} \\ \max(S_{t_0} - \texttt{OPS1}, 0) & \text{else if} \quad \texttt{OPTP} = \text{'C'} \\ \max(\texttt{OPS1} - S_{t_0}, 0) & \text{else if} \quad \texttt{OPTP} = \text{'P'} \\ \max(S_{t_0} - \texttt{OPS1}, 0) & \text{else} \\ \quad + \max(\texttt{OPS2} - S_{t_0}, 0) & \end{cases}$	with $S_{t_0} = O^{rf}(U^{ca}(Child, MOC), t_0)$
Prf		Same as PAM
Led		Same as PAM

OPTNS: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Pos}_{t^+} = \begin{cases} \max(S_t - \texttt{OPS1}, 0) & \text{if} \quad \texttt{OPTP} = \text{'C'} \\ \max(\texttt{OPS1} - S_t, 0) & \text{else if} \quad \texttt{OPTP} = \text{'P'} \\ \max(S_t - \texttt{OPS1}, 0) & \text{else} \\ \quad + \max(\texttt{OPS2} - S_t, 0) & \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $S_t = O^{rf}(U^{ca}(Child, MOC), t)$
PRD	$D(\mathbf{Prf}_{t^-})(-1)\texttt{PPRD}$	STF_PRD_STK()
TD	$D(\mathbf{Prf}_{t^-})\texttt{PTD}$	STF_TD_STK()
XD	0.0	$\mathbf{Pos}_{t^+} = \begin{cases} \max(S_t - \texttt{OPS1}, 0) & \text{if} \quad \texttt{OPTP} = \text{'C'} \\ \max(\texttt{OPS1} - S_t, 0) & \text{else if} \quad \texttt{OPTP} = \text{'P'} \\ \max(S_t - \texttt{OPS1}, 0) & \text{else} \\ \quad + \max(\texttt{OPS2} - S_t, 0) & \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $S_t = O^{rf}(U^{ca}(Child, MOC), t)$
STD	$D(\mathbf{Prf}_{t^-})R(\texttt{CNTRL})\mathbf{Pos}_{t^-}$	$\mathbf{Pos}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
CD	POF_CD_PAM()	STF_CD_STK()

9.16. **FUTUR: Future.**

FUTUR: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
XD		Same as OPTNS
STD		Same as OPTNS
CD		Same as PAM

FUTUR: State Variables Initialization

State	Initialization per t_0	Comments
Pos	$\mathbf{Pos}_{t_0} = \begin{cases} 0.0 & \text{if} \quad t_0 \leq \texttt{MD} \\ S_{t_0} - \texttt{PFUT} & \text{else} \end{cases}$	with $S_{t_0} = O^{rf}(U^{ca}(Child, MOC), t_0)$
Prf		Same as PAM
Led		Same as PAM

FUTUR: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Pos}_{t^+} = S_t - \texttt{PFUT}$ $\mathbf{Led}_{t^+} = t$ with $S_t = O^{rf}(U^{ca}(Child, MOC), t)$
PRD	POF_PRD_OPTNS()	STF_PRD_STK()
TD	POF_TD_OPTNS()	STF_TD_STK()
XD	POF_XD_OPTNS()	$\mathbf{Pos}_{t^+} = S_t - \texttt{PFUT}$ $\mathbf{Led}_{t^+} = t$ with $S_t = O^{rf}(U^{ca}(Child, MOC), t)$
STD	POF_STD_OPTNS()	STF_STD_OPTNS()
CD	POF_CD_PAM()	STF_CD_STK()

9.17. **CEG: Credit Enhancement Guarantee.**

CEG: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
FP		Same as PAM
XD	$t^{XD} = \begin{cases} U^{ca}(\texttt{CECC}_1, \texttt{NPD}) & \text{if} \quad U^{ca}(\texttt{CECC}_1, \texttt{NPD}) \neq \varnothing \\ \tau(O^{ev}(i, t_0)) & \text{else if} \quad O^{ev}(i, t_0) \neq \varnothing \\ & \quad \wedge \tau(O^{ev}(i, t_0)) < \mathbf{Tmd}_{t_0} \\ \varnothing & \text{else} \end{cases}$	with $i = \begin{cases} \texttt{CECLEI} & \text{if} \quad \texttt{CECLEI} \neq \varnothing \\ U^{ca}(\texttt{CECC}_1, LEICP) & \text{else} \end{cases}$
STD	$t^{STD} = t^{XD}$	
MD	$t^{MD} = \mathbf{Tmd}_{t_0}$	
CD		Same as PAM

CEG: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{CECLEI} \neq \varnothing \\ \max(\tau(\{e_t^k\})) & \text{else} \end{cases}$	with $\{e_t^k\} = \{e_t^{k,1}\} \cup \{e_t^{k,2}\} \cup ... \cup \{e_t^{k,n}\}$ $\{e_t^{k,i}\} = U^{ev}(\texttt{CECC}_i, t_0 \mid \{\texttt{CNTRL} = RPA\})$ $n = \mid \texttt{CECC} \mid$
Fac	$\mathbf{Fac}_{t_0} = \begin{cases} 0.0 & \text{if} \quad \texttt{FER} = \varnothing \\ \texttt{FEAC} & \text{else if} \quad \texttt{FEAC} \neq \varnothing \\ nY(t^-, t_0)\texttt{FER} & \text{else if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t_0)}{Y(t^{FP-}, t^{FP+})} \times \texttt{FER} & \text{else} \end{cases}$	with $n = \begin{cases} \texttt{NT} & \text{if} \quad \texttt{NT} \neq \varnothing \\ \sum_{i=1}^{\lvert\texttt{CECC}\rvert} U^{sv}(\texttt{CECC}_i, \texttt{CDD}, \mathbf{Nvl} \mid \{x\}) & \text{else} \end{cases}$ $x = \text{'}\texttt{CNTRL}\text{=RPA'}$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
Pos	$\mathbf{Pos}_{t_0} = \begin{cases} 0.0 & \text{if} \quad t_0 \geq \mathbf{Tmd}_{t_0} \\ \texttt{CECV} \times \texttt{NT} & \text{else if} \quad \texttt{NT} \neq \varnothing \\ \texttt{CECV} \sum_{i=1}^{\lvert\texttt{CECC}\rvert} n_i & \text{else} \end{cases}$	with $n_i = \begin{cases} U^{sv}(\texttt{CECC}_i, t_0, \mathbf{Nvl} \mid \{x\}) & \text{if} \quad \texttt{CEGE} = NO \\ U^{sv}(\texttt{CECC}_i, t_0, \mathbf{Nvl} \mid \{x\}) & \text{else if} \quad \texttt{CEGE} = NI \\ \quad +U^{sv}(\texttt{CECC}_i, t_0, \mathbf{Nac} \mid \{x\}) & \\ O^{rf}(U^{ca}(\texttt{CECC}_i, MOC), t_0) & \text{else} \end{cases}$ and $x = \text{'}\texttt{CNTRL}\text{=RPA'}$
Prf		Same as PAM
Led		Same as PAM

CEG: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Pos}_{t^+} = \begin{cases} 0.0 & \text{if } \; t \geq \mathbf{Tmd}_{t_0} \\ \texttt{CECV} \times \texttt{NT} & \text{else if } \; \texttt{NT} \neq \varnothing \\ \texttt{CECV} \sum_{i=1}^{\lvert\texttt{CECC}\rvert} n_i & \text{else} \end{cases}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \; \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{if } \; \texttt{CEGE} = NO \\ U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{else if } \; \texttt{CEGE} = NI \\ \quad +U^{sv}(\texttt{CECC}_i, t, \mathbf{Nac} \mid \{x\}) & \\ O^{rf}(U^{ca}(\texttt{CECC}_i, MOC), t) & \text{else} \end{cases}$ $n = \begin{cases} \texttt{NT} & \text{if } \; \texttt{NT} \neq \varnothing \\ \sum_{i=1}^{\lvert\texttt{CECC}\rvert} U^{sv}(\texttt{CECC}_i, \texttt{CDD}, \mathbf{Nvl} \mid \{x\}) & \text{else} \end{cases}$ $x = \text{'}\texttt{CNTRL}\text{=RPA'}$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PRD	POF_PRD_STK()	STF_PRD_STK()
FP	$\begin{cases} \texttt{FER} & \text{if } \; \texttt{FEB} = A \\ \mathbf{Fac}_{t^-} + nY(t^-, t)\texttt{FER} & \text{else} \end{cases}$ with $n = \begin{cases} \texttt{NT} & \text{if } \; \texttt{NT} \neq \varnothing \\ \sum_{i=1}^{\lvert\texttt{CECC}\rvert} U^{sv}(\texttt{CECC}_i, \texttt{CDD}, \mathbf{Nvl} \mid \{x\}) & \text{else} \end{cases}$	$\mathbf{Fac}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
XD	POF_XD_OPTNS()	$\mathbf{Pos}_{t^+} = \begin{cases} \texttt{CECV} \times \texttt{NT} & \text{if } \; \texttt{NT} \neq \varnothing \\ \texttt{CECV} \sum_{i=1}^{\lvert\texttt{CECC}\rvert} n_i & \text{else} \end{cases}$ $\mathbf{Fac}_{t^+} = \begin{cases} \mathbf{Fac}_{t^-} + Y(\mathbf{Led}_{t^-}, t)\mathbf{Nvl}_{t^-}\texttt{FER} & \text{if } \; \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{if } \; \texttt{CEGE} = NO \\ U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{else if } \; \texttt{CEGE} = NI \\ \quad +U^{sv}(\texttt{CECC}_i, t, \mathbf{Nac} \mid \{x\}) & \\ O^{rf}(U^{ca}(\texttt{CECC}_i, MOC), t) & \text{else} \end{cases}$ $n = \begin{cases} \texttt{NT} & \text{if } \; \texttt{NT} \neq \varnothing \\ \sum_{i=1}^{\lvert\texttt{CECC}\rvert} U^{sv}(\texttt{CECC}_i, \texttt{CDD}, \mathbf{Nvl} \mid \{x\}) & \text{else} \end{cases}$ $x = \text{'}\texttt{CNTRL}\text{=RPA'}$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
STD	POF_STD_OPTNS()	STF_STD_OPTNS()
MD	0.0	$\mathbf{Pos}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
CD	POF_CD_PAM()	STF_CD_STK()

9.18. **CEC: Credit Enhancement Collateral.**

CEC: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
XD		Same as CEG
STD		Same as CEG
MD		Same as CEG
CD		Same as PAM

CEC: State Variables Initialization

State	Initialization per t_0	Comments
Tmd	$\mathbf{Tmd}_{t_0} = \max(\tau(\{e_t^k\}))$	with $\{e_t^k\} = \{e_t^{k,1}\} \cup \{e_t^{k,2}\} \cup ... \cup \{e_t^{k,n}\}$ $\{e_t^{k,i}\} = U^{ev}(\texttt{CECC}_i, t_0 \mid \{\texttt{CNTRL} = RPA\})$ $n = \mid \texttt{CECC} \mid$
Pos	$\mathbf{Pos}_{t_0} = \begin{cases} 0.0 & \text{if} \quad t_0 \geq \mathbf{Tmd}_{t_0} \\ \min\left(\sum_{i=1}^{\lvert\texttt{CECE}\rvert} v_i, \texttt{CECV} \sum_{i=1}^{\lvert\texttt{CECC}\rvert} n_i\right) & \text{else} \end{cases}$	with $n_i = \begin{cases} U^{sv}(\texttt{CECC}_i, t_0, \mathbf{Nvl} \mid \{x\}) & \text{if} \quad \texttt{CEGE} = NO \\ U^{sv}(\texttt{CECC}_i, t_0, \mathbf{Nvl} \mid \{x\}) & \text{else if} \quad \texttt{CEGE} = NI \\ \quad +U^{sv}(\texttt{CECC}_i, t_0, \mathbf{Nac} \mid \{x\}) & \\ O^{rf}(U^{ca}(\texttt{CECC}_i, MOC), t_0) & \text{else} \end{cases}$ $v_i = O^{rf}(U^{ca}(\texttt{CECE}_i, MOC), t_0)$ $x =$ '$\texttt{CNTRL}$=RPA'
Prf		Same as PAM
Led		Same as PAM

CEC: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Pos}_{t^+} = \begin{cases} 0.0 & \text{if} \quad t \geq \mathbf{Tmd}_{t_0} \\ \min\left(\sum_{i=1}^{\lvert\texttt{CECE}\rvert} v_i, \texttt{CECV} \sum_{i=1}^{\lvert\texttt{CECC}\rvert} n_i\right) & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{if} \quad \texttt{CEGE} = NO \\ U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{else if} \quad \texttt{CEGE} = NI \\ \quad +U^{sv}(\texttt{CECC}_i, t, \mathbf{Nac} \mid \{x\}) & \\ O^{rf}(U^{ca}(\texttt{CECC}_i, MOC), t) & \text{else} \end{cases}$ $v_i = O^{rf}(U^{ca}(\texttt{CECE}_i, MOC), t)$ $x =$ '$\texttt{CNTRL}$=RPA'
XD	POF_XD_OPTNS()	$\mathbf{Pos}_{t^+} = \begin{cases} 0.0 & \text{if} \quad t \geq \mathbf{Tmd}_{t_0} \\ \min\left(\sum_{i=1}^{\lvert\texttt{CECE}\rvert} v_i, \texttt{CECV} \sum_{i=1}^{\lvert\texttt{CECC}\rvert} n_i\right) & \text{else} \end{cases}$ $\mathbf{Led}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{if} \quad \texttt{CEGE} = NO \\ U^{sv}(\texttt{CECC}_i, t, \mathbf{Nvl} \mid \{x\}) & \text{else if} \quad \texttt{CEGE} = NI \\ \quad +U^{sv}(\texttt{CECC}_i, t, \mathbf{Nac} \mid \{x\}) & \\ O^{rf}(U^{ca}(\texttt{CECC}_i, MOC), t) & \text{else} \end{cases}$ $v_i = O^{rf}(U^{ca}(\texttt{CECE}_i, MOC), t)$ $x =$ '$\texttt{CNTRL}$=RPA'
STD	POF_STD_OPTNS()	STF_STD_OPTNS()
MD	0.0	$\mathbf{Pos}_{t^+} = 0.0$ $\mathbf{Led}_{t^+} = t$
CD	POF_CD_PAM()	STF_CD_STK()

